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ORIENTAÇÕES PARA O PLANEAMENTO E IMPLEMENTAÇÃO DE UM PROGRAMA DE FORMAÇÃO DE PROFESSORES EM REGIME DE APRENDIZAGEM MISTA: UMA COMUNIDADE DE PERSPECTIVA PRÁTICA

GUIDELINES FOR PLANNING AND IMPLEMENTING A BLENDED LEARNING TEACHER EDUCATION PROGRAM: A COMMUNITY OF PRACTICE PERSPECTIVE

DIRECTRICES PARA LA PLANIFICACIÓN Y EJECUCIÓN DE UN PROGRAMA DE FORMACIÓN DEL PROFESORADO DE APRENDIZAJE COMBINADO: UNA COMUNIDAD DE PERSPECTIVA PRÁCTICA

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Abstract

During the Covid-19 pandemic emerged a substantial educational debate about professional tools and skills for distance learning. This paper is part of a Ph.D. research, developed before and during the pandemic situation, about mathematics teacher education in distance education. Using the situated learning theory, focused on the community of practices concept, we developed an e-research with a mixed-method design. The main aim of the research was to characterize pre-service mathematics teachers' e-learning and b-learning practices in distance education in Brazil. Data collected included two phases: an online survey sent to pre-service teachers from seven mathematics teacher education programs in distance education in Brazil, five professors' interviews and program projects analysis from these universities, covering all five geographic regions (first phase); and seven pre-service teachers' interviews from the initial mathematics teachers' program at the Federal University of Tocantins (second phase). As a direct consequence of the data analysis, in both phases, and the discussions about the conception and function of the communities of practice in this context, guidelines were formulated aiming to contribute to the consolidation of teacher education programs in distance education. This paper will focus on the characterization and detailing of these guidelines.

Keywords: teacher education, distance education, communities of practice, b-learning, guidelines

Resumo

Durante a pandemia de Covid-19 surgiu um grande debate educacional sobre ferramentas e habilidades profissionais para o ensino a distância. Este artigo é parte de uma pesquisa de doutorado realizada antes e durante a pandemia sobre a formação de professores de matemática na educação a distância. Utilizando a teoria da aprendizagem situada, focada no conceito de comunidade de práticas, desenvolvemos uma e-research com um design misto. O objetivo principal da investigação foi caracterizar as práticas de e-learning e b-learning de professores de matemática em formação a distância no Brasil. Os dados coletados incluíram duas fases: um questionário online enviado para sete programas de formação de professores de matemática a distância, entrevistas com cinco professores e análise dos projetos de programas dessas universidades, abrangendo todas as cinco regiões geográficas do país (primeira fase); e sete entrevistas com futuros professores de matemática da Universidade Federal do Tocantins (segunda fase). Como consequência direta da análise dos dados, em ambas as fases, e das discussões sobre a concepção e função das comunidades de prática na formação de professores, foram formuladas *guidelines* visando contribuir para a consolidação dos programas de formação de professores na modalidade a distância. Este trabalho concentrar-se-á na caracterização e detalhamento das *guidelines* propostas.

Palavras-chave: formação de professores, educação a distância, comunidades de prática, b-learning, *guidelines*

Resumen

Durante la pandemia de Covid-19 surgió un debate educativo sustancial sobre herramientas y habilidades profesionales para el aprendizaje a distancia. Este artículo es parte de una tesis doctoral en investigación, desarrollada antes y durante la situación de pandemia, sobre la formación del profesorado de matemáticas en la educación a distancia. Utilizando la teoría del aprendizaje situado, centrada en el concepto de comunidad de prácticas, desarrollamos una e-investigación con un diseño de método mixto. El objetivo principal de la investigación fue caracterizar las prácticas de e-learning y b-learning de los futuros profesores de matemáticas en la educación a distancia en Brasil. Los datos recopilados incluyeron dos fases: una encuesta en línea enviada a futuros profesores de siete programas de formación de profesores de matemáticas en educación a distancia en Brasil, cinco entrevistas a profesores y análisis de proyectos de programas de estas universidades, que cubren las cinco regiones geográficas (primera fase); y siete entrevistas a futuros profesores del programa inicial de profesores de matemáticas de la Universidad Federal de Tocantins (segunda fase). Como consecuencia directa del análisis de los datos, en ambas fases, y de las discusiones sobre la concepción y función de las comunidades de práctica en este contexto, fueron formuladas directrices con el objetivo de contribuir a la consolidación de los programas de formación de profesores en educación a distancia. Este documento se centrará en la caracterización y detalle de estas directrices.

Palabras clave: formación docente, educación a distancia, comunidades de práctica, b-learning, *guidelines*

INTRODUCTION

Authors often call today's society by "knowledge society" (Hargreaves, 2003), "network society" (Castells, 2005; Hoadley & Kali, 2019), "digital society" (Monteiro, Moreira & Lencastre, 2015), or "learning society" (Coutinho, 2010). Usually, these nomenclatures represent our society after the emergence and diffusion of microcomputers and the Internet. The DataReportal study (2021) mentions that the number of internet

users rose by more than 316 million between January 2020 and January 2021. When studying only social network users, the rise is even more impressive.

Even with a great effort from the research in Education (Hargreaves & Giles, 2003; Collins & Halverson, 2009; Coutinho, 2010; Pedro & Matos, 2015; Pedro, 2017) in the study and proposals for reviewing the traditional face-to-face teaching model and with all perceived impact throughout the Covid-19 pandemic on schools and universities (Flores & Gago, 2020; Prata-Linhares et al., 2020; Chirinda et al., 2021), this education model is still, in turn, strongly influenced by the economic, social and cultural events of the 17th and 18th centuries, which were intended to respond to the needs of industrial society.

When we started the doctoral project in 2017, we did not imagine that three years later we would be experiencing the Covid-19 pandemic, nor did we foresee that our daily lives would be transformed in such a drastic way. What became popularized as the "new normal" reflected a new way of relating to each other, the way we developed our work, our family relationships, our studies, our shopping, our consumption of art, among other types of behavior. Some rooms were transformed into offices and mothers and fathers reconciled work with daily care for children at home, with remote classes. Society has changed, even temporarily. It changed unevenly, especially in the Brazilian context, but not only. We lost a lot but also developed adaptation strategies that a state of emergency like this requires. Consequently, as citizens, we reflected on our everyday lives and our relationship with the world we inhabit.

As part of this society, schools are also (re)organized and (re)invented. School reinvention reflected, as well, in the debate on distance education (DE) and, above all, on the tools that help teachers to develop online classes. Even without knowing what awaits us in 2020, to approach the object of study of this investigation, the distance mathematics teachers' education, we proposed, at the beginning of the Ph.D., the development of an e-research, i.e., the data for this research were collected online, before and during the pandemic situation.

There has been an increase in Brazilian investigations aimed at understanding distance learning experiences, especially with the exponential increase in distance teacher education programs since the creation of the "Open University of Brazil" (UAB) system in 2006. In this sense, we sought, in the literature review, to map the publications on experiences in Mathematics Education and DE from the year 2011, since these UAB programs were already more consolidated and, consequently, the investigations on these more widespread.

A systematic review of the literature (SRL) on the concept of DE was developed (Prates & Matos, 2020), in its different models, analyzing previous investigations focused on Mathematics Education (basic education, initial or continuing teacher training). This review showed us, among other things, a trend in studies on teacher education and the importance of the principles that govern communities of practice in this type of training. The studies, analyzed throughout the SRL, also showed us that when the creation of a learning community in online environments is identified, collaborative, collective work, partnerships, and dialogues contribute, in an effective way, to the practices of mathematics teachers and teachers-to-be.

In this sense, it is important to invest in distance learning course models that favor a perspective of collaborative work from the planning, conception, and development of the course. Creating conditions for students to feel involved in online courses and, eventually, create learning communities, are also features that deserve special attention from investigations in the field of mathematics education and distance learning.

In a previous paper (Prates & Matos, 2021), we showed that pre-service mathematics teachers highlight collaboration between colleagues as the main reason for their stay throughout the course. In other words, the collaboration between peers is important for the maintenance of communities of practice in the initial mathematics teacher's education. Despite this, pre-service teachers, in this same study, point to a lack of activities, proposed by the course, that support collaborative or even group work. We decided to create guidelines to support mathematics teacher education programs in DE.

In this paper, we will first present the background literature of the community of practice (CoP) focusing on the studies relating to mathematics teacher education that inspired us to collect data and propose the guidelines. Afterwards, we will show the methodology used in the Ph.D. research, to illustrate how the e-research in a mixed method design was developed. In the end, we will present some results with the detailed categorization of the guideline built.

1. BACKGROUND LITERATURE

Over the last two decades, several studies have been produced relating the concept of community of practice, created by Lave & Wenger (1991) and improved by Wenger (1998), with the training of mathematics teachers, whether initial or continuous. Goos (2014) defines communities of practice in mathematics teacher education as something that is based on "[...] the theory of learning as social participation, in which teacher learning and development are conceptualized as increasing participation in social practices that develop an identity as a teacher" (p. 82). In this process of developing an identity as a teacher, prospective pre-service mathematics teachers can be understood as "newcomers" in the mathematics teacher community.

Investigations in Mathematics Education relate the aspect of collaboration with the constitution of (virtual) communities of practice (Llinares & Krainer, 2006; Miskulin et al., 2011; Hjalmarson, 2017). Llinares and Krainer (2006) carried out a literature review based on investigations produced by members of the PME (Psychology of Mathematics Education) and observed that "The factors influencing teacher development (reflection and collaboration), are instruments that teacher educators consider when designing teacher education programs, but they are not professional development objectives in themselves" (pp. 443-444). These authors are concerned with how to understand the relationship between these instruments (collaboration and reflection) and changes in teachers' practices.

Hjalmarson (2015, p. 282) points out, for example, that the main challenge for distance teacher education programs is "[...] to provide online opportunities using improved technological tools while maintaining the quality of the programs and remaining consistent with the best practices for mathematics teacher education and professional development". By studying her practice in a mathematics teacher training course, this author relates the development of online or face-to-face teacher education to the concept of CoP and suggests that aspects such as community and collaboration are directly related to the teachers' engagement in the course.

The work by Miskulin et al. (2011, p. 178) also shows a very close relationship between collaboration and learning outcomes in virtual environments, noting that by learning together, in a virtual community, students "[...] have the opportunity to transpose and deepen their learning experiences, experiencing new ideas, sharing them with the group, and receiving critical and constructive feedback". In this investigation, the students were also university professors of different mathematics teacher programs in Brazil. These teachers participated in online courses on the use of Information and Communication Technologies (ICT) in Mathematics Education. One of the "dimensions" that the authors contemplated in this investigation, of utmost importance to us, was "the teacher education processes in communities". The authors believe that

> the process of forming a community assumes as a premise the constitution of a group of people, of teachers who teach Mathematics, [...] who seek to reflect on issues of different natures, guided by common objectives and interests that involve, often, their practices related to teaching work. When this process takes place through a virtual learning community, in which common interests and objectives, actions, dialogue, reflective discourse, shared experiences and collaboration are articulated, resulting in implications for learning and teaching, perhaps a possible redefinition will occur in teaching practice, constituted in this collective and

> social interlocution. [...] Dimensions such as collaboration, interaction, repertoires of practices, and reflections are highlighted as essential to the redefinition of teaching practice. (p. 179)

In this perspective, the creation of a community, in the context of mathematics teacher education, not only enriches the theoretical repertoire of teachers or future teachers but also helps in teaching practices.

In the context of distance education, some Brazilian authors (e.g., Miskulin et al., 2011; Almeida & Borba, 2015; Silva & Schimiguel, 2016) suggest that, by necessarily using digital technologies, DE courses favor collaboration between peers. Almeida and Borba (2015) highlight that "environments such as the discussion forum allow the production of knowledge in a collective and collaborative way" (p. 53). In line with this observation, Silva and Schimiguel (2016) mention that with the paradigm shift experienced in the educational system with the advent of information and communication technologies, "collaboration between peers is expanded, which enables the development of creativity, innovation and thought focused on solving problems" (p. 43). However, the cited works did not observe these actions in practice. The first is a literature review on the distance learning courses in Mathematics and the second is a theoretical work on problem-based learning (PBL), Statistical Education, and DE. The research conducted by Miskulin et al. (2011) deals with what they call "collaboration in virtuality". The professors participating in the study were in continuing education in an online context. The researchers claim that "collaborating, sharing narratives, facts, problems, experiences, anxieties, expectations and learning stories reveal aspects of each one's teaching practice, and this fact can be of fundamental importance in the training process of teachers who teach Mathematics" (Miskulin et al., 2011, p. 176). In this context, they mention practices that can be developed during online courses and that favor the formative process of mathematics teachers. The authors also state that collaboration "contributes to socially shared learning and reduces student isolation, which can occur in virtual environments" (p. 177).

Still, on the feeling of community, Du, Liu & Brown (2010) define, based on a literature review, four key elements of an online learning community, namely: *interactivity*, *collaboration*, *trusting relationship*s, and *communication media*. The authors conclude that interactivity and collaboration can and should be encouraged among students both in small groups and with work on projects, for example. For them, trusting relationships are fostered from the social interaction between community members and, for this, communication media that have higher levels of social immediacy will provide more opportunities for interaction between students, allowing them to form a more cohesive community (Du, Liu & Brown, 2010, p. 71).

When we characterize the practices of pre-service teachers in a mathematics teacher education program in DE, we seek, from the demands imposed by society, to identify the elements of a community of practice that, in some way, meet these demands. Do teacher education programs consider the aspects described above and which, in some way, are associated with a new way of relating to each other as a society? Do they consider the social characteristics of the "lived world"? These and other questions led us to think about general guidelines for the planning and execution of a distance teacher training course.

2. METHODOLOGY

The methodological design was developed in two distinct phases: in the first phase, a convergent design followed; however, this design resulted in the planning of a second phase of data collection and analysis. We will call it an explanatory sequential convergent design. This name was inspired by the three main designs of mixed methods described by Creswell & Creswell (2018, p. 300), as the second phase was planned and executed after analyzing the data collected in the first phase, which, despite having two qualitative components, the main source of data from pre-service teachers (the focus of the second phase) was quantitative (via an online survey).

2.1. Participants and the context

During the first data collection phase, we sought to understand more broadly how the initial mathematics teacher programs in distance education are theoretically and methodologically organized. In addition, we needed to select a group of pre-service teachers, from the seven programs analyzed, to participate in the interviews about the routines and activities of the program (phase two). This selection was mainly made through the online survey about the pre-service teachers' perception concerning the model of the teacher education program in DE that they attend, i.e., which pre-service teachers feel more involved and satisfied with the course? Which groups of pre-service teachers have greater (or lesser) difficulty in developing the courses? Or which group (or groups) of pre-service teachers feel(s) that this type of program favors learning? During the first phase, interviews were also carried out with the coordinators of the five courses in question.

Thus, in Table 1 below, the participating universities in the different phases of the investigation are found, including the testing and validation of the instruments.

Table 1

Instruments used and participating universities by phase.

Phase	Data collection instruments	Universities	Data source
Validation	Online Survey	UFS	33
	Interview	UFS	1
First	Documents: Pedagogical Projects	UFT; UFSC; UFAL; UFSJ; Unemat	5
	Online Survey	UFRR; UFT; UFSC; UFAL; UFPE; UFSJ; Unemat	144
	Coordinator's Interviews	UFT; UFSC; UFSJ; UFAL; Unemat	5
Second	Pre-service teachers' Interviews	UFT	7

Note. UFAL: Federal University of Alagoas; UFPE: Federal University of Pernambuco; UFRR: Federal University of Roraima; UFS: Federal University of Sergipe; UFSC: Federal University of Santa Catarina; UFSJ: Federal University of São João-del-Rei; UFT: Federal University of Tocantins; Unemat: State University of Mato Grosso.

2.2. Data source and analysis method

Due to the need to identify the different models adopted for mathematics teachers' education in DE and to select a group of pre-service teachers for the second stage of the investigation, the data collection process took place in two distinct phases. In addition, during the first phase, we resorted to three different sources of data: documents, interviews with coordinators and pre-service teachers, and pre-service teachers' responses to the survey. Due to this diversity of sources and instruments, there is also a diversity of data collection and analysis processes. In addition, it is worth mentioning that, as these are different materials, the analysis methods also differ depending on the data collected. For the data from the surveys, for example, statistical analyzes were used: Cronbach's alpha, factorial analysis, and ANOVA. We used the software SPSS for statistical analysis. For the analysis of the pedagogical projects of the courses and the interviews with the pre-service teachers and professors, a content analysis was used (Bardin, 1977), with, in the first case, the projects, a first approach of the documental analysis and only then proceeding with the thematic categorical analysis. In this case, to help us in the content analysis, we used the software

NVivo. When collecting qualitative data, it is natural that we have an enormous amount of descriptive information that needs to be organized and reduced (Coutinho, 2011).

This is what happened with the data from the three qualitative sources of the investigation: documents, interviews with professors, and pre-service teachers. In this way, we went through the three phases of the content analysis method (Coutinho, 2011): pre-analysis, material exploration, and treatment of results (inference and interpretation). However, despite using the same data analysis technique, there is a difference in the categorization procedures. In the case of documents and interviews with coordinators, the analysis categories were defined later. In the interviews with the pre-service teachers, the categories were previously defined, although two more categories emerged in the process of exploring the material.

3. GUIDELINES DESCRIPTION

While undertaking the data analysis, we noticed that the shared practices are restricted to a reduced group of pre-service teachers. In addition, future mathematics teachers have little access, or almost none, to the arenas of more maturity of mathematics teachers in basic education, or even of the training professors or tutors of the education program in which they are enrolled. In this sense, so that future mathematics teachers could access these arenas, the design of a distance teacher education program must consider the new paradigm of communication and access to the content that we live together today. As an example, in the analysis of the interview data, all pre-service teachers mentioned the importance of using the *Whatsapp* group and *YouTube* videos as tools for online communication and access to content that facilitate the development of a shared repertoire by them. *Whatsapp* groups were not created and encouraged by professors or course coordinators. These groups are initiatives of pre-service teachers themselves and have become an important part of online practices throughout the course. In the case of *YouTube* videos, even though some professors indicate them as complementary material, there is still a concern to prepare video lessons and provide them as the main source of content. In the speeches of pre-service teachers, both about *Whatsapp* groups and *YouTube* videos, we can infer that these tools are extremely present and fundamental in their practices. They are strategies created or perfected by them to achieve the central objective of completing their academic careers successfully. Polin (2010), when discussing the use of technologies in communities of practice, mentions some tools that can support online teachings, such as blogs, wikis, and social networks (p. 168). The author adds that these tools also "explicitly represent notions of continuous modification of content through participation, a crucial counterpoint to typical student perceptions of university knowledge as static, dated, and isolated" (p. 169). This counterpoint cited by the author was what we noticed both in the use of *Whatsapp* groups and *Youtube* by the pre-service teachers interviewed in our investigation. As they created different and, to a certain extent, transgressive strategies, they opposed the idea of university knowledge as isolated, static, and dated.

In this way, we believe it is essential that the planning of training courses for distance mathematics teachers consider the possibility of continuous modification of the curriculum and support tools. This modification should be based on the participation of all those involved in the community: be it the pre-service mathematics teachers, the professors and coordinators of the teacher education program, the basic education teachers who accompany the pre-service teachers throughout their practices, and who directly support the development of a course of this nature. Given what has already been exposed, we propose some general guidelines for the planning and development of an initial mathematics teachers' education program. Considering the nature of the training course, i.e., the fact that it is a distance learning course, but not only. The proposed guidelines also start from the fact that these pre-service teachers are training mathematics teachers in basic education, which, at least in the Brazilian context, is essentially face-to-face. For this, we divided the guidelines into three large blocks: the format of the teacher education program, the curriculum, and the work and assessment methodologies.

3.1. The format of the teacher education program

A teacher training course with 100% of activities online, or with only the final assessments in person, as is the case of the courses studied throughout this investigation, misses something extremely important from the perspective of situated learning: contact with the professional field. Because, despite having training in DE, future mathematics teachers will teach in face-to-face teaching and ideally, contact with the field of work, that is, with the school and, even more, with the classroom and with the practices of mathematics teachers in face-to-face schools, starting in the first semester of the course. In this way, the first guideline is that being in DE, opt for a blended-learning course, i.e., part of the activities is developed at a distance, through learning platforms and online communication tools, and face-to-face part, at the support centers in each city. The suggested workload is at least 50% of face-to-face and/or synchronous meetings, that is, professors and pre-service teachers connected at the same time. And this contact with the school since the beginning of training leads us to discuss the next dimension.

3.2. The curriculum

As already mentioned in previous studies (Ávila & Santos, 2014; Gatti, 2014) and confirmed in the present investigation, the curriculum of a DE course cannot be a reproduction of face-to-face curricula. In this sense, we propose that this be thought out in a way that favors the objectives determined by the courses, which, in general, can be summarized as training mathematics teachers to work in basic education, meeting the demands of society. In this way, we propose that curricular integration is a way of associating university education, both at the level of i) school, classroom, already mature professional practices (such as basic education teachers) and ii) among course pre-service teachers, that is, activities that integrate peers throughout training; and also, iii) between the different components of the training course, be it the specific content of mathematics or the pedagogical and technological component. From the first moment of the course, pre-service teachers must attend some basic education school to understand how it works: the physical structure (laboratories, libraries, classrooms, open areas, canteens, etc.), number of employees, regulations, among other aspects. In this sense, the curriculum should start from this contact with the school. It is this internship experience from the first moment that will guide the other activities throughout the course.

In addition, and as this research outcomes show, there is a need for pre-service teachers, and the very idea of autonomy in DE (Holmberg, 1995), for the curriculum to be more flexible and for future teachers to be free to choose not only the ideal time to study, but also what they want to study according to their academic path. For example, if the curriculum is designed in study modules, pre-service teachers can, depending on the context they find at school, choose to complete a certain module earlier or later than expected. If a certain pre-service teacher arrived at the school and the teacher who welcomed him for the internship was working on the geometry content, the curriculum must be flexible enough so that he can choose to study the disciplines that support him in this observation, monitoring, or intervention at school.

The example given by Professor António Nóvoa (2018, p. 21-22) on the new curriculum of the Faculty of Medicine at Harvard University can perfectly be applied to the context of training distance mathematics teachers. It is divided into four main pillars: “collaborative learning models”; “individualized training courses”; “a long connection with the profession”; and the “appreciation of work outside the university, with periods of work with the population”. This previously suggested integration is related to collaborative learning models, to the extent that pre-service teachers, working in groups in the chosen subject modules, can share their personal experiences in schools, exchange learning scenarios and lived situations, and reflect together on these experiences. But it is also related to the pillar that deals with the long connection with the profession. As already mentioned, it is essential that, from the perspective of situated learning, future mathematics teachers are close to the school and the classroom from the very beginning of the

course. The school is outside the university; it would then be the period in which they would have to work with the population. We suggest that this contact be gradual and that, at first, future mathematics teachers can observe, and get to know the reality and functioning of a school, beyond what is already known as students they once were, in participation more peripheral. This participation can (and should) increase over time, until in the final years of the course, future teachers can experience the pedagogical residency more completely, as teachers responsible for a class; later, for a longer period until they reach a whole semester in the same class.

Finally, what we find in the medical curriculum at Harvard University as individualized training paths is directly linked to the curricular flexibility that we mentioned earlier. Pre-service teachers must choose what one is going to study depending on what one is in contact with at school. The internship starting in the first semester brings the advantage that, over four or five years of training, the future mathematics teacher will have time to attend different schools, classrooms, and teaching levels. With the flexibility in choosing the curricular components, pre-service teachers can adapt their modules to enrich both their internship and discussions with their peers throughout their studies at the university.

3.3. Work and assessment methodologies

A work methodology design is essential to implement the development of a curriculum that prioritizes integration, at different levels, flexibility, and personalization in teacher education. In other words, the demand for weekly completed lists of exercises, as we found in data analysis, will hardly help in the integration of different contents and the solution of real problems experienced in schools. Group work cannot be an exception in teacher education, as it currently happens in the studied programs. Future teachers should, from the beginning of the course, share experiences among themselves, and between themselves and the professors. As we noticed throughout the data analyses, future mathematics teachers help each other even when they are instructed to do the exercises individually. In this sense, this strategy, already used by them, can and should be implemented as a collaborative learning model.

The so-called "active learning methodologies" that include different strategies such as "problem-based learning", "project-based learning", "case-based learning", or "inquiry-based learning", for example, can be a way of supporting and inspiration this connection between the activities developed in schools and those developed in the university. Thus, pre-service teachers can, for example, from real cases, experienced in the internship's field schools, detect problems related to teaching and learning mathematics, share these cases and problems in meetings, either in person or online, throughout the course, and find possible solutions to these problems. Future teachers can also, at the end of this process, develop projects that are related to the cases and problems studied.

These methodological strategies can also be a fertile field for uniting teaching and research. We should also emphasize here the importance of stimulating and presenting research throughout the distance initial mathematics teachers' education programs. Research should be part of the pre-service teacher's entire academic trajectory and not just at specific times.

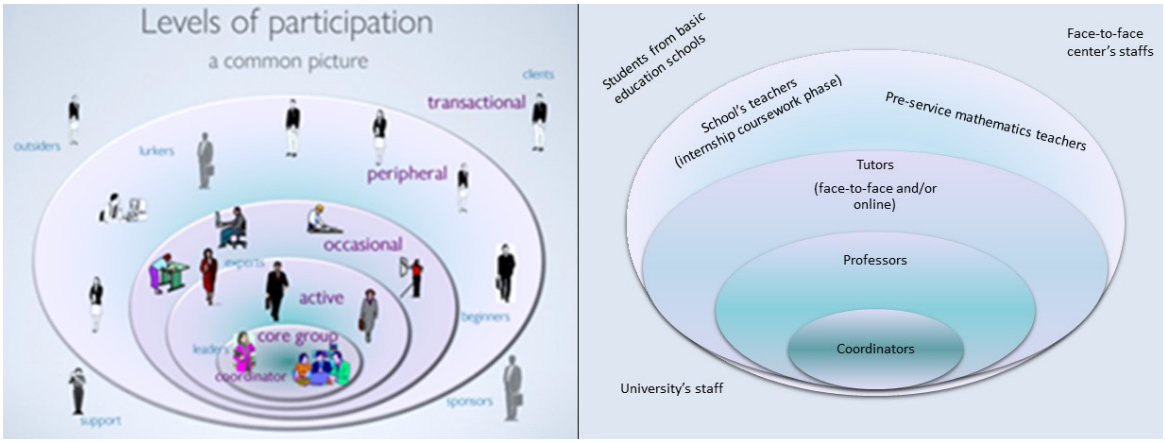
Finally, we propose that the evaluation of future mathematics teachers in training should not be limited to a written test by content at the end of a specific period, as it currently happens in the studied programs. The focus should be on both the variety of assessments (written, oral, project reports, presentations, videos, etc.) and the format (individual, group, self-assessments, peer reviews, etc.). The most important thing here is that the pre-service teacher is accompanied throughout his/her training process and that he/she can present to colleagues, professors, and tutors what he/she has developed throughout a project or a teaching module.

Even more explicitly than in an architecture or engineering course, for example, pre-service teachers in a mathematics teacher education program will necessarily have the same jobas the course's professors. In other words, after four or five years of training, they will be mathematics teachers. Even if we are talking about teacher education for basic education, the practice of higher education professors (related to lesson planning, classes, assessments, etc.) is much closer to the practice of basic education teachers than of architects or engineers. Some of these future distance mathematics teachers can (and, as we saw in the interviews, some already do) hold a master's and a doctorate and become professional colleagues (i.e., members of the same community of practice as their professors). The teaching profession is very familiar to all of us. Llinares and Krainer (2006) consider the learning of mathematics teachers as "[...] a lifelong learning process which starts with one's own experiences of mathematics teaching from the perspective of a student, or even with mathematical activities before schooling" (p. 429). In this way, all these pre-service teachers who are graduating to be mathematics teachers have already created representations about this job well before entering the course.

It is hoped that, with these guidelines for designing a training course for mathematics teachers at a distance, all people who are somehow engaged in this training will feel part of a community, sharing the same domain of interest and developing shared practices. As future mathematics teachers are seen by those who idealize, propose and act in these training courses, as newcomers in the community of practice of mathematics teachers (Figure 1), support for engagement, imagination, and alignment work can be facilitated in the development of courses. Therefore, through fostering these three ways of belonging, which we believe that is quite feasible within the context studied in this research, future mathematics teachers will be able to build a more solid professional identity.

Figure 1

Levels of participation in a CoP proposed by Wenger-Trayner (2021) X Our proposal of levels of participation in a community of practice in the mathematics teachers' education program in distance learning.



Note. The image on the left side was found at: https://wenger-trayner.com/project/levels-of-participation/

FINAL REMARKS

We understand the difficulty of modifying some paradigms so rooted in the educational system. In the process of rethinking initial teacher education, there are also legal obstacles to reformulating the curriculum or an evaluation process that deviates from the traditional individual written test, for example. Based on the results of the Ph.D. research we designed guidelines to propose a distance learning program that reflects the theoretical perspective adopted: the community of practice. However, we hope that they

are not seen as something static and imposing. They are guidelines to inspire new pedagogical projects, proposals for activities, and/or revisions of the legislation that regulates teacher education in Brazil. Furthermore, we would like to point out that these guidelines also consider existing actions. The Institutional Program for Teaching Initiation Scholarships (PIBID) and professional master's degrees in Brazil show us that the articulation between universities and basic education schools, in the initial and continuing teachers' education, is possible. The guidelines described above are aimed not only at expanding the articulation that is already being made promptly, in supervised internships or in projects, but also to make it the starting point for all teacher education.

Finally, we reiterate the importance of the theoretical framework of the CoP to analyze the practices in e-learning and b-learning of future mathematics teachers and to rethink and recreate how these teacher education programs are currently registered. The CoP, in the analysis of this context, sheds light on the processes of construction, development, and evaluation in mathematics teachers' education, which are so based on the processes experienced in face-to-face courses. Furthermore, this theory helps us to understand learning as an integral part of social practice (Lave & Wenger, 1991). Future works can be based on the analytical framework of the CoP, look at the specific activities of teacher education programs in DE, and try to understand which of them and which tools have been used that favor and encourage collaborative practices and participation in the community of practice. In addition, the future studies can also plan, propose, and develop teacher education programs considering the guidelines created to investigate and analyze the results of the training experience.

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